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Engaging With Mathematics: Exploring Different Learning Environments at the Elementary School Level

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Abstract

This article examines the "effectiveness" of school-level mathematics and explores how teachers' instruction, within classrooms, contributes to an effective learning. By using multiple data sources, including classroom observations and teacher interviews, we found that students from "effective schools" receive more intellectual support than students in "typical schools" and benefit from better learning environments overall. We also found that the overall intellectual quality differs as per students' ability level, favoring students of "lower ability" when they are enrolled in effective schools. Teacher interviews suggest that teachers from effective schools tend to hold higher expectations for low-ability students than in typical schools and know how to adapt to them by adopting individualized instruction to meet their learning needs.

Keywords

classroom learning environment; effective schools; elementary schools; mathematics; typical schools

1. Introduction

School effectiveness research (SER) has grown rapidly over the past four decades, partly because it seeks to answer fundamental questions about education—answers that lie at the core of why students in some schools perform better than in others—and promote the development of so-called "effective schools." Effective schools are commonly understood as ones in which students perform better than in other schools on most academic subjects, generally as measured via standardized tests (Clewell et al., 2007); typical schools are those where students score average or below-average on standardized tests. From its inception, SER focused on identifying non-instructional factors that could influence the "effectiveness" of a school, factors that relate, among other things, to the environment, organization, resources, and culture of a school. This continues to be a feature of SER more broadly. Purkey and Smith's (1983) early review helped define the concept of effective school by providing a list of characteristics that add extra value to students' learning experiences: school organization and structure (see also Bedard & Do, 2005), external support, leadership and staff stability, safety, and cultural elements. Features like school management (Cheng, 2022) and parent involvement (Pelemo, 2022) have also been investigated for their association with students' learning outcomes and are some of the non-instructional factors that have been shown to contribute to school-level effectiveness.

More recently, research on classroom teaching has explored the importance of classroom-level, instructional factors on effectiveness and there is a growing acceptance that the best way to improve ineffective schools is by providing them with better teachers and high-quality classroom instruction (Waxman et al., 2007). Research on teaching practices has also advanced our understanding of the factors that are more effective for students' learning of mathematics. Numerous studies have documented that teaching practices are positively associated with student mathematical achievement and play a critical role in academic success (Osborne, 2021; Tan & Dimmock, 2022; Yu & Singh, 2018). Using a generic observational framework, Zhu and Kaiser (2022) found that teachers' social support and teaching quality positively impacted students' mathematical performance, whereas classroom management did not have a significant direct impact. Likewise, using an observational framework focused on teaching practices that are specific to teaching mathematics, Merritt et al. (2017) found that teachers' use of multiple representations, vocabulary-building techniques, and attention to individuals' understanding were prevalent practices in high-gain classrooms.

These findings suggest the need for considering both cross-classroom and within-classroom features when examining school effectiveness. Yet, as previously mentioned, most studies on teaching quality have not been situated within the broader context of school effectiveness. Reynolds (2010) believed that the separation of SER and teaching quality research might be ignoring school-level conditions and arrangements that can contribute to an effective instruction within classrooms. In the case of mathematics specifically, researchers have found that classroom-level factors, such as students' perception of "in-classroom" learning environments (Waxman et al., 2021) and structural teaching, time on mathematical tasks, and curriculum quality (Scheerens, 2016), significantly contribute to school effectiveness. Within-classroom features such as students' gender and ability levels have also been found to influence teachers' instructional practices. For instance, Nurmi's (2012) meta-analysis found that teachers tend to give more attention to students they expect to perform well compared to those they expect to perform poorly. Desimone and Long (2010) reported that teachers in schools located in disadvantaged socioeconomic areas spent less teaching time on low-achieving students. These two studies suggest it might be necessary to consider within-classroom factors when examining the relationship between classroom instruction and school effectiveness.

Most research on mathematics instructional quality (Merritt et al., 2017; Osborne, 2021; Zhu & Kaiser, 2022) has not been within the context of school effectiveness. Limited attention has been paid simultaneously to school-level, classroom-level, and student characteristics. Most SER research has focused on one or, at the most, two out of three instructional-level factors that are known to impact student mathematics achievement: students, classrooms, and schools. Extant classroom-level studies have focused on general teacher behaviors or classroom instruction (Charalambous & Praetorius, 2018; Zhu & Kaiser, 2022). In this article, we examine all three factors simultaneously and use systematic, mathematics-specific observation scales.

We aligned students' learning behaviors with teachers' instructional practices in what we refer to as "classroom learning environment." A strict focus on teacher practices misses the impact that students' behavior has on their own learning. Teaching practices can only be effective if students engage with them. Thus, classroom learning environment refers to a coordination of within-classroom instruction that includes teacher behavior (Fisher et al., 2005) as well as teacher–student and/or student–student interactions (Zedan, 2010).

This article also looks at the "intellectual quality" of environments specific to the learning of mathematics. By "intellectual quality" we mean the depth to which mathematics is treated in a given classroom, the understanding thereof that is evident in the classroom, and the range of students' engagement with mathematics (see Newmann et al., 1996; Secada & Lee, 2000). The following four research questions guide this study:

1. Are there any significant differences in the learning environment and student engagement with mathematics between "highly effective" and "typical" urban schools?
2. Are students' experiences of mathematical learning environments and engagement different based on students' ability levels, and whether students' ability levels interact with school-level effectiveness?
3. Are students' experiences of mathematical learning environments and engagement different based on students' sex, and do students' sex interact with school-level effectiveness and student ability?
4. (exploratory) What underlying teacher perceptions about mathematics, their students, and/or teaching might shed light on these findings?

1.1. Quality of Mathematics Instruction Within Effective Schools

Some studies have provided evidence that teachers in effective schools consistently display "effective teaching" behaviors. Using high-inference classroom observations of 65 reading teachers from effective schools versus 51 teachers from ineffective schools, Teddlie et al. (1989) found that reading teachers in effective schools exhibit more "student time on task," frequent presentation of new materials, high expectations for students, positive reinforcement, and a friendly learning environment. Teddlie and Reynolds (2000) distinguished effective schools from less effective schools in the following ways: Math teachers in the more effective schools were more likely to adopt student-centered activities and small-group instruction, their students were encouraged to use mathematical manipulations and problem-solving techniques, and the teachers were more likely to extend the curriculum by adding relevant information from varied sources beyond textbooks. Teacher-directed and whole-class instruction were primarily used in less effective schools.

Based on the survey and interviews of 523 teachers from two school districts (10 effective schools and six typical schools), Clewell et al. (2007) found that highly effective school teachers reported holding higher expectations of their students, spending more time working on the mathematical content, and developing their own materials for mathematical instruction. Aside from differences in teacher practices, highly effective schools had a higher proportion of qualified and experienced teachers, and provided more professional development opportunities for their teachers than their counterpart schools. The same study relied on direct observation of teachers' classrooms, embedded students within those classrooms, and, in an exploratory manner, related instructional practices to teacher beliefs and perceptions.

Studies have investigated the differences in students' learning behaviors and their perceptions of classroom learning environments between effective and ineffective schools. Waxman et al.'s (2021) observations of 947 fourth- and fifth-grade students' behaviors during reading and mathematics found that students in effective schools were much more engaged in tasks, perceived learning environments as more positive, and had more time to interact with their teachers. Students enrolled in average and ineffective schools were less engaged in tasks, more distracted, and had much lower perceptions of their learning environments.

1.2. Effective Urban Schools

Research on the effective school goes back to Edmonds (1979), who sought to provide direction for improving the education of students enrolled in urban schools in the United States by focusing on school-level characteristics of effective schools. Subsequent work in urban settings consistently reported similar findings. Muijs et al.'s (2004) review lists the following factors as those that most impact schools in socioeconomically disadvantaged areas: leadership capacity, a positive school culture, continuous professional development of teachers and administrators, teaching and learning, and external support. A set of Canadian case studies that examined 12 secondary schools in low-income settings drew the conclusion that high expectations for all students, a clear focus on academic achievement, and supportive structure and services contributed to the success of their students (Raham, 2004). In a case study of three English schools with high proportions of students from minority ethnic groups qualifying for free school meals, Engel et al. (2010) identified the ethos of inclusion as central to the success of schools catering to students from challenging environments. They argued that inclusion went beyond classroom strategies and practices; rather, it permeated the entire school and affected schools in every aspect of how they functioned.

1.3. Student Factors Within an Effective School Setting

Students' socio-demographic characteristics can be linked to their cultural backgrounds in relation to classroom instruction, either positively as resources for instruction or negatively by providing fewer learning opportunities during instruction. The importance of differentiating classroom instruction arises from the fact that students differ from one another in terms of prior knowledge, interests, motivation, learning needs, and learning styles (Dowson & McInerney, 2003). By adapting instruction to individual students' needs, teachers increase students' chances of success and help to reduce the learning gaps among students. Creemers and Kyriakides (2007) studied the extent to which teachers give more instructional opportunities and instruction to students who need them. J. B. Smith et al. (2001) found that low-achieving, economically disadvantaged students benefited from the interactive instruction that emphasized teachers' guidance and encouraged students' explanation and discussion of mathematical concepts. Ketterlin-Geller et al. (2008) found that additional exposure to the core mathematical concepts and practices increased the achievement gains of state-standardized tests for low-achieving students. Unfortunately, numerous studies have found that low-achieving students from disadvantaged socioeconomic areas tend to get teachers who spend less time on instruction and provide less conceptual and more procedural instruction compared with their counterparts in high SES settings (Desimone & Long, 2010; J. B. Smith et al., 2001; T. M. Smith et al., 2005).

Early gender-based teaching research found that teachers favored male over female students in the quantity and quality of their interactions (Garrahy, 2001; Wimer et al., 2001), a trend that more recent research continues to confirm regarding teachers' perception and expectations of students' abilities and performance.

Riegle-Crumb and Humphries (2012), for instance, found that high school math teachers perceived boys (especially white boys) as more capable than girls, just as Tiedemann (2002), a decade earlier, found that middle-school teachers considered boys to possess higher mathematical abilities and profit more from additional efforts than girls. Teachers may also respond differently to boys and girls based on students' abilities: Dukmak (2009) found that teachers initiated more interactions with high-achieving boys than with high-achieving girls; yet, they initiated more interactions with low-achieving girls than with low-achieving boys.

2. Conceptual Framework for Classroom Environments

2.1. Simultaneous Focus on Teachers and Students

Teaching and learning are social practices that are dependent on context; student learning does not happen in isolation (Lemke, 2001). Both teachers and students co-create and support classroom learning environments. A teacher may engage in practices that focus on in-depth treatment of a mathematical topic. But without broad student uptake based on those practices, students are unlikely to learn the content in much depth. Similarly, teacher practices may focus on developing students' understanding of mathematical topics and/or on mathematical analysis (higher-order thinking). Without students' co-participation in those practices, students' understanding and application of analysis are unlikely. Our focus on learning environments as co-constructed by teachers and students highlights that teachers and their students simultaneously shape and contribute to the intellectual quality of the classroom's learning environment.

Socio-constructivist learning theory provides a foundation for this view of learning since it portrays learning as an active process, where learners actively construct their mathematical ways of knowing as they participate in the classroom community (Cobb & Yackel, 1996). Students are not passively receiving knowledge from teachers' transmissions (e.g., pretending to listen to lectures), but they are actively involved in the learning processes. By building on one another's contributions, teachers and students create a rich learning environment. Based on this theoretical perspective, we assessed the quality of learning environments not by focusing on a single aspect of the classroom, such as teacher quality (Jong et al., 2004), instructional tasks, and task implementation (Stigler & Hiebert, 2004), or classroom artifacts (Clare & Aschbacher, 2001). We adopted an interactional lens that focused on the relationship among individual students, teachers, and the context within which that student learning took place.

2.2. Things Matter for Mathematical Learning

We focused on three aspects in mathematical learning: (a) depth in the treatment of mathematical content, (b) level of focus in developing conceptual understanding, and (c) support for engaging students in mathematical analysis (high-order thinking). These three aspects are consistent with the *Principles to Actions: Ensuring Mathematical Success for All*, a document by the National Council of Teachers of Mathematics (NCTM) that provides research-based descriptions of instructional/teaching practices in support of students' learning of mathematics (NCTM, 2014). In addition to developing a conceptual understanding of key mathematical ideas, the NCTM also recommends a flexible use of procedures and strategies to engage students in mathematical practices, including reasoning, problem-solving, and communicating. Our standards were derived from what Newmann (1996) called "authentic learning," which argued that teaching and learning should lead students to a deep understanding of concepts and relationships in contexts that involve real-world problems. Newmann and Weglage (1993) provided five authentic instructional practices that align with these learning goals, of which we adopted the following three: higher-order thinking, depth of knowledge, and social support for student achievement.

Mathematical tasks provide the basis for classroom teaching and learning. Students should spend a substantial portion of instructional time focused on challenging tasks and that will keep them deeply engaged in mathematics content, so that they can achieve rigorous mathematical learning goals (Cobb & Wilhelm, 2022; Sullivan et al., 2015). Sullivan et al. (2015) and Sullivan and Mornane (2014) have worked with groups of primary and secondary teachers to support their efforts to convert challenging mathematical tasks into classroom lessons. Challenging tasks allow students opportunities to engage with important mathematical ideas, to choose their strategies, goals, and levels of accessing the task, and to extend their knowledge and thinking in new ways. Sullivan and Mornane (2014) stressed that challenging tasks are those that help students build networks and connections between ideas with which students are familiar. Stein et al. (2008) stated that challenging tasks are cognitively demanding and promote conceptual understanding and the development of thinking, reasoning, and problem-solving skills.

Students learning mathematics with understanding is a critical feature of the principles and standards for school mathematics (NCTM, 2014). Learning mathematics with understanding involves students in understanding mathematical ideas and procedures and making connections among ideas (Stylianides & Stylianides, 2007). These connections facilitate the transfer of prior knowledge to novel situations. Memorizing facts or procedures without understanding results in fragile and non-transferable learning. In this case, students' knowledge may be shallow, thin, or superficial and cannot be transferred to other contexts since the mere mastery of facts and procedures does not necessarily imply mathematical proficiency (NCTM, 2014). The NCTM clarifies that effective teaching of mathematics builds fluency with procedures on the foundation of conceptual understanding; thus, conceptual understanding and procedural fluency are essential and integrated components of mathematical proficiency.

Higher-order thinking skills refer to the ability to analyze, evaluate, and create, based on Bloom's (1956) taxonomy. Unfortunately, people have used the term when discussing mathematics in the classroom with no attachment to actual math content. To highlight that mathematics is the content of higher-order thinking, we used the term "mathematical analysis." Mathematical analysis involves searching for mathematical patterns, making mathematical conjectures, and justifying them (Secada & Lee, 2000). Mathematical analysis also includes organizing, synthesizing, evaluating, arguing, hypothesizing, describing patterns, and making models or simulations. In all these cases, the "content of the thinking" is mathematics.

3. Method

In our research, we followed an overall SER logic, i.e., we identified schools that were considered highly effective in mathematics based on multi-year results from their state mathematics assessments and compared them to schools whose performance was typical for their respective districts. We observed multiple sessions of classroom instruction spread over the Fall and Spring of a given academic year: Fall observations focused on whole-classroom instruction, while Spring observations followed individual students of the same classroom. Observers did not know which schools had been identified as highly effective, nor which observed students had been identified in terms of their mathematical ability. We assumed that classroom instruction is relatively stable across a given school year, as Newmann et al. (1996) found in their unpublished data analysis.

3.1. Criteria for the Selection of Schools

Data for this study were extracted from a larger study that aimed at identifying the different characteristics, practices, and policies between effective schools and typical schools around disadvantaged communities (see Clewell et al., 2007; Secada & Lee, 2000). Two districts, President City Area and Arbor City, were selected for serving a large population of low-income and minority students. The majority of students (85%) in the President City Area were Hispanic, 11% were white, and 3% were African American. In the Arbor City district, most district students (65%) were African American, 16% were white, and 13% were Hispanic/Latino (Secada & Lee, 2000).

The criteria for identifying highly effective schools in each district were based on state-wide standardized mathematics tests at the fourth-grade level across a minimum of three years prior to the study. Highly effective schools were those where a higher percentage of fourth graders demonstrated at least a basic level of expertise in mathematics, with a significant number of students exhibiting high levels of proficiency. Ninety percent or more of fourth graders passing their mathematics tests, and 50% or more of fourth graders scoring proficient or higher were used as the criteria in the President City area. As national science tests were also available in Arbor City, both mathematics and science tests were used as the criteria for selecting effective schools; that is, schools with 66% or more of fourth graders scoring basic or above-basic levels, and 30% or more of fourth graders scoring proficient or above-proficient levels were considered as the potential highly effective schools. Based on the criteria, five elementary schools were selected as the highly effective schools in each district. Three elementary schools were selected as the counterpart typical schools; they scored less well on the same tests but were closely matched in terms of student demographics in the same district (Clewell & Campbell, 2007).

3.2. Instruments and Data Collection

A high-inference classroom observational instrument was developed and used in the present study, derived from the observation scales developed by the Center for Organization and Restructuring of Schools (https://www.centerforcsri.org) to measure instructional quality (Newmann et al., 1996). These scales provide a systematic observation schedule to measure overall classroom instruction beyond dyadic interactions. This instrument is grounded in the idea that authentic assessment should emphasize certain practices (or instructions) to help students achieve an in-depth understanding of academic subjects (Newmann, 1996). The individual observation scales (from 1 to 5) and their brief descriptions are provided below:

1. Measuring intellectual support: To what extent is the classroom learning environment characterized by an atmosphere of high academic expectations for all students, coupled with mutual respect and support among students?
2. Measuring depth of knowledge and student understanding: To what extent is mathematical knowledge treated deeply in the class? To what extent is knowledge treated in a shallow and superficial manner?
3. Measuring mathematical analysis: To what extent do students use mathematical analysis?

4. Student engagement was measured by students' on-task behaviors that signal a serious psychological investment in class work.

Each trained on-site team consisted of two mathematics education specialists. During the Fall observation, they observed the teachers' mathematics classroom teaching twice. Observers did not know which schools were highly effective versus typical.

The original whole-class observational instrument was modified to follow individual students during the Spring observation. This shift in emphasis recognizes the relationship between individual students and the context in which students' learning is taking place. This change allowed the unit of analysis to be individual students. During each of the two classroom visits, the same three students were observed: One had been listed by the teachers as being of high ability, another was considered of medium ability, and a final one of low ability. Each student was observed twice for between five to 10 minutes per lesson. Student sex was counter-balanced at the school level. Two observations per student, per visit, and two visits resulted in four observations per student in total. Spring observers did not know which schools were "effective," nor did they know any student's ability level. Table A in the Supplementary File presents the number of observations in both Fall and Spring.

3.3. Teacher Interviews

Semi-structured interviews were conducted to understand teachers' conceptions, knowledge, and teaching beliefs involving student diversity. The following topics were considered: (a) general reactions to their lessons, (b) conceptions of the nature of mathematics, (c) conceptions of student diversity, and (d) conceptions of teaching mathematics to students of different ability levels. Fall and Spring interviews followed similar protocols. Spring interviews added some questions related to teachers' expectations, evaluations, and teaching to the three observed students. Each interview lasted about 45 minutes to one hour. In the Spring, 15 mathematics teachers from the highly effective schools and 11 mathematics teachers from the typical schools were interviewed. For this study, we analyzed those interviews wherein teachers talked about the three individually observed students.

3.4. Data Analysis

The mixed-methods sequential explanatory approach began with quantitative, followed by qualitative analyses (Creswell et al., 2003). The quantitative phase used advanced multi-level modeling (Reynolds, 2010) to examine the differences in classroom learning environments between typical and highly effective schools. The qualitative phase focused on the interview data to help explain and elaborate on the quantitative results obtained in the first phase. The quantitative results guide the case selection for the qualitative analyses.

This reanalysis of an extant database taken from Secada and Lee (2000) used more advanced methodology and statistical tools than were available at that time, which allowed us to pursue the research questions listed at the start of this manuscript. The quantitative analysis used a three-level model to analyze the classroom learning environment: Students were embedded within their fourth-grade classrooms; classrooms were embedded within their schools; and schools were categorized as either effective or typical. Student-level factors are students' demographic information, including sex (female vs. male) and ability (high ability, medium ability, and low ability). In our study, low-ability students were defined as those perceived by their teachers as performing below average in mathematical learning. Gender is a social construction that, at present, is highly contested. Hence, we are relying on the biological term of sex and use that terminology.

To establish that a three-level model is suitable and relevant for this analysis—it is significantly better than the two- or single-level model (traditional regression analysis)—we used a likelihood ratio test (LRT). The three-level model is better than the two-level model without the school level for the dimensions of intellectual support and intellectual quality, and better than the one-level model for all the dimensions, except for student engagement. The three-level model fits the data better than models with fewer levels.

The Fall classroom observations provide the classroom-level factors to examine whether the classroom learning environment from the same teachers was consistent across terms. The Spring observations of intellectual support, student understanding, mathematical analysis, the intellectual quality of the learning environment (averaged from the three prior subscales), and student engagement were used as the dependent variables.

All the models were estimated using restricted maximum likelihood (REML), which is commonly considered a viable option for obtaining accurate parameter estimates with small sample sizes (Kenward & RogeR, 1997) in the R environment using the lme4 package (Bates & Sarkar, 2007). Data analysis proceeded in the two steps: First, unconditional models without any predictors were run to check the variance explained at the classroom and schools; then, full models including student-level (ability and sex), classroom-level (Fall observations), and school-level (school types) variables, interaction between school types with students' sex, ability, and Fall observation, were run. We ran full models instead of adding factors level by level because our research questions involve all three levels simultaneously.

In the qualitative phase, we selected representative teachers based on their quantitative instructional quality scores to examine how their perceptions and self-reported practices aligned with observed classroom practices. We employed open-ended coding for teachers' expectations, teachers' evaluation of students' abilities to learn mathematics, specific teaching strategies for low-ability students, and the challenges of teaching those students. Particular attention was given to aspects that corresponded with the observed classroom outcomes.

4. Results

4.1. Descriptive Statistical Analysis

Table 1 reports the number of students and teachers, student-level means, and standard deviation (SD) for Spring classroom observations based on the school types. Students from effective schools demonstrated slightly higher mean scores for depth of knowledge, mathematical analysis, and student engagement than students from typical schools. The mean differences in intellectual support and instructional quality between typical and effective schools were relatively larger, suggesting that school types may contribute to the differences in those two dimensions. Table 2 presents the three-level analyses results on Spring classroom learning environments. The discussion of results by research questions is based on Table 2.

Table 1. Student-level means (SD) for Spring classroom observations by school types.

Type of school	No. of students	No. of teachers	Intellectual support	Depth of knowledge	Mathematical analysis	Instructional quality	Student engagement
Typical	33	11	2.49(0.64)	1.28(0.37)	1.14(0.23)	4.93(0.79)	3.37(0.67)
Effective	49	26	2.84(0.44)	1.51(0.52)	1.32(0.37)	5.68(1.09)	3.40(0.85)

Note: Instructional quality is the sum of the three sub-scales.

Table 2. Beta values (and standard error) from multi-level model analyses on classroom learning environments.

Variables	Intellectual supports		Depth and understanding		Math analysis		Intellectual quality		Student engagement	
	β	*SE*	*β*	*SE*	*β*	*SE*	*β*	*SE*	*β*	*SE*
School-level										
Effective	3.85**	1.16	1.50	1.01	−0.04	0.99	4.16**	1.29	−0.32	1.12
Teacher-level										
Fall Intellectual	0.96**	0.34								
Fall Understanding			0.25	0.63						
Fall Analysis					−0.71	0.06				
Fall Instruction							1.17*	0.51		
Fall Engagement									0.23	0.17
Student-level										
High ability	0.34	0.34	−0.11	0.35	0.89	0.45	0.38	0.30	0.18	0.49
Male	−0.83*	0.34	−0.80*	0.36	−0.22	0.42	−0.84*	0.35	−0.24	0.45
Interaction										
Effective × High ability	−0.72	0.36	−0.63	0.37	−0.42	0.49	−0.79*	0.38	−0.04	0.52
Effective × Male	0.17	0.31	0.42	0.33	0.24	0.42	0.30	0.33	−0.09	0.44
High ability × Male	0.71	0.39	0.84*	0.41	−0.42	0.49	0.67	0.40	−0.01	0.55

Notes: ** $p < 0.01$, * $p < 0.05$; typical schools are the reference group for school type, low-ability students for ability, and female for student sex.

4.2. Differences Across School Types

Table 2 shows that the differences in overall intellectual quality were significant ($\beta = 4.16$, $SE = 1.29$, $p < 0.01$) across school types, favoring effective schools. However, no significant differences in student engagement were found. Although students in both effective and typical schools are engaged to a similar extent in mathematical learning, students in effective schools experience and contribute towards a higher intellectual quality of mathematical learning environments than their peers in typical schools.

The differences in intellectual support were significant across school types, favoring the effective schools ($\beta = 3.85$, $SE = 1.16$, $p < 0.01$), suggesting that an important reason that students in effective schools could have experienced and contributed towards a higher intellectual quality was that students experienced and contributed to a learning environment that was characterized by an atmosphere of high academic standards for all students coupled with mutual respect and support among teachers and students.

4.3. Differences Across Ability Levels and School Types

We found no significant differences in student engagement across students' ability levels or across school types (Table 2). Across both effective and typical schools, low-ability students engage comparably to high-ability students, and students from effective schools engage comparably to their peers at the same ability level as those in typical schools. Hence, students' engagement in their mathematics classrooms seems unaffected by their ability level or by the effectiveness of their school.

Figure 1, based on the interaction for intellectual quality (Table 2), shows that high-ability students in both typical and effective schools get similar mathematical learning experiences in classrooms. So, what makes some schools effective? The differences in the overall intellectual quality were significant in effective schools ($\beta = -0.79$, $SE = 0.38$, $p < 0.05$) between students' ability levels, favoring low-ability students. This indicates that low-ability students in highly effective schools experienced instruction of higher intellectual quality than did their high-ability peers, suggesting that teachers in effective schools allocate their instructional resources (which shape the learning environment) based on students' needs.

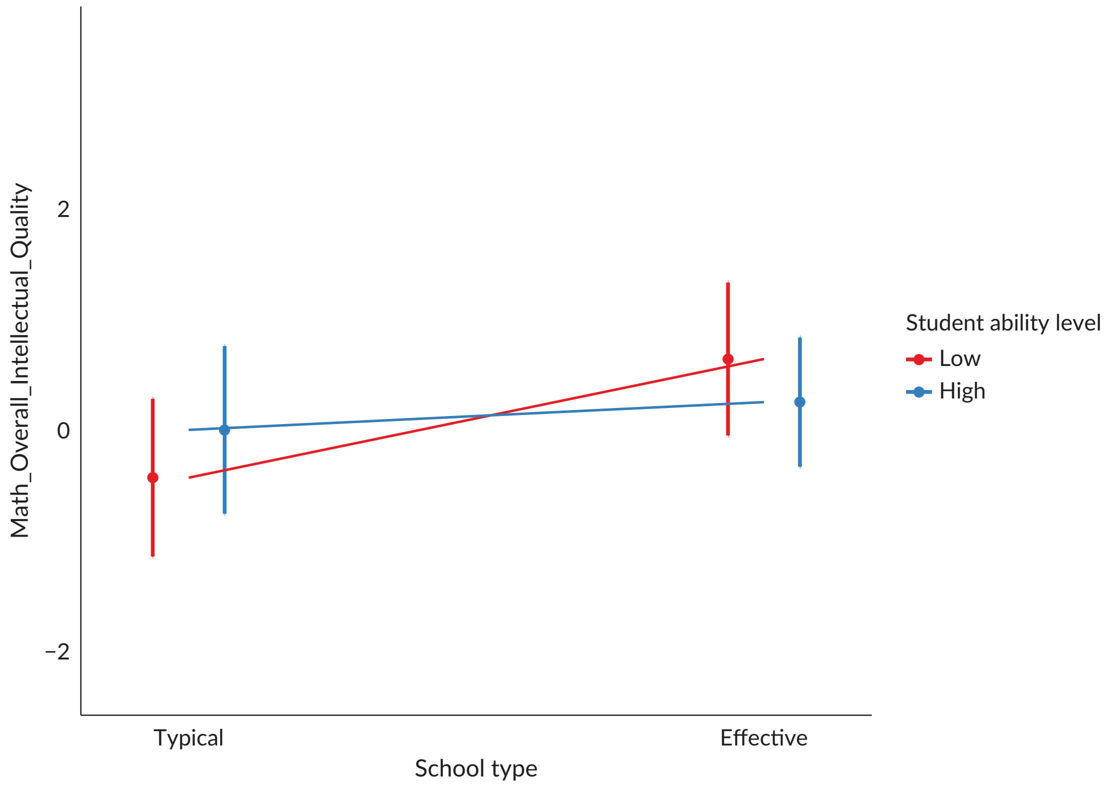

Figure 1. The interaction between students' ability and school types for the overall intellectual quality.

4.4. Differences Across Students' Sex, Ability Levels, and School Types

Previous research has found that mathematics instruction favors and benefits male students over female students (Garrahy, 2001; Wimer et al., 2001). However, our results indicate the contrary. Differences in the intellectual quality of the classroom learning environment between male and female students were significant ($\beta = -0.84$, $SE = 0.35$, $p < 0.05$), favoring female students with no interaction (Table 2, $\beta = 0.30$, *SE* = 0.33, $p < 0.05$). Female students in both effective and typical schools experience and contribute towards a higher intellectual quality learning environment than their male peers.

Furthermore, significant differences in intellectual support and depth and understanding ($\beta = -0.83$, $SE = 0.34$, $p < 0.05$; $\beta = -0.80$, $SE = 0.36$, $p < 0.05$) were found between female and male students across both types of schools, favoring female students. These findings suggest that some specific ways in which female students experience a higher quality mathematics learning environment in their classrooms are because teachers and their female students work together towards (a) conveying and accepting high academic standards and mutual respect and support and (b) focusing on deeper engagement in mathematical knowledge, to a greater extent than do male students.

Figure 2 shows the contrast between students' ability level and sex for students' depth of knowledge and understanding, to further understand the nuances of our results in the context of students' ability levels. Previous literature states that high-ability male students are the most benefited group, as far as instructional quality is concerned, in mathematics classrooms because teachers perceive them to have greater mathematical abilities than their female peers (Riegle-Crumb & Humphries, 2012). However, we found that low-ability and high-ability female students experienced deeper engagement in mathematical knowledge to a similar extent as high-ability male students. This is an important finding because it shows that teacher bias towards high-ability males during mathematical instruction may have been attenuated by the female students responding more deeply to their teachers' efforts.

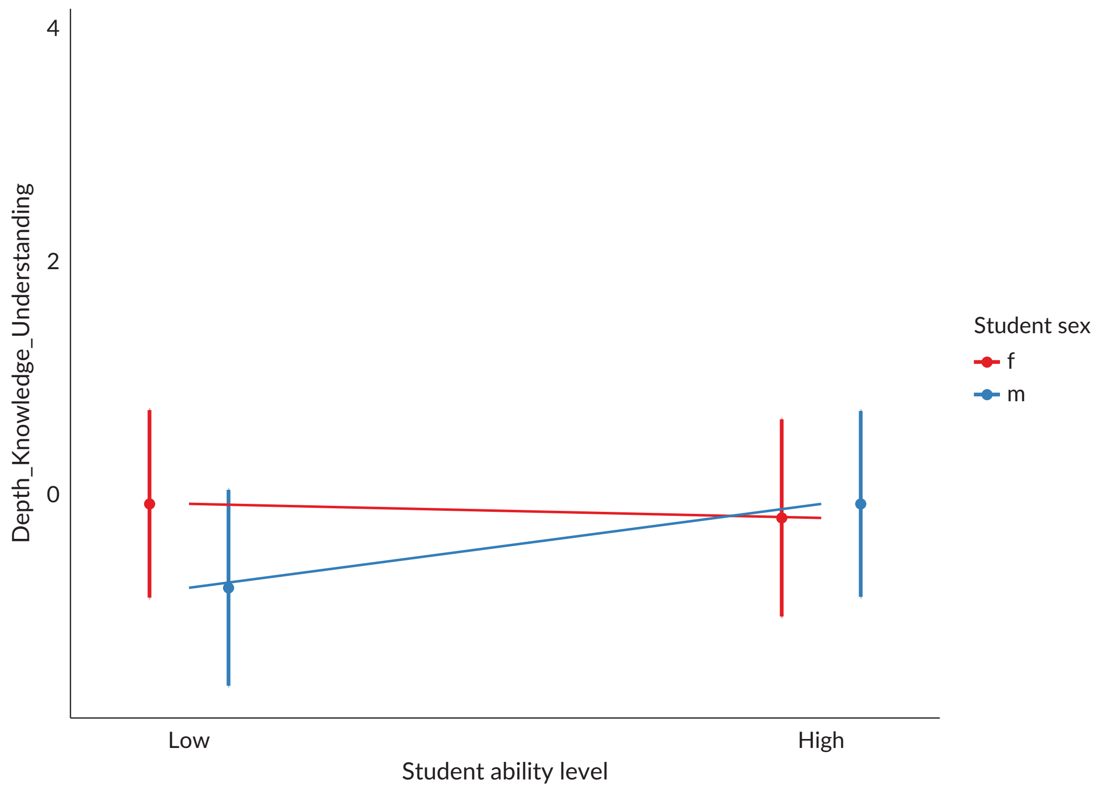

Figure 2. The interaction between student ability levels and sex for students' depth of knowledge and understanding.

Additionally, we found that male students with lower ability levels experienced the least depth of knowledge and understanding compared to all other peer groups. This finding is consistent with previous literature that low-ability male students were perceived to have discipline issues and lower mathematical abilities by their teachers and, as a result, received instruction aligned with lower academic expectations (Straehler-Pohl et al., 2014).

No significant differences in student engagement were found between male and female students in both typical and effective schools. This suggests that student engagement in schools is not affected by students' sex, suggesting that the engagement of all students in mathematics classrooms is comparable.

4.5. Teachers' Underlying Perceptions and Beliefs

On average, teachers in effective schools provided a higher intellectual quality in their learning environments to low-ability students compared to high-ability students. Whereas, in typical schools, teachers provided greater intellectual quality to high-ability students as compared to low-ability students. In our qualitative analyses, we selected teachers whose classrooms provided a similar pattern of intellectual quality to low-and high-ability students based on the type of schools in which they taught. In the effective schools, three teachers satisfied the two criteria. After examining whether teachers discussed low-ability students, two teachers, Mrs. B (2 points to the low-ability student and 1.58 points to the high-ability student) and Mrs. R (1.7 points to the low-ability student and 1.67 points to the high-ability student), were selected as the representative teachers. In typical schools, two teachers, Mrs. A (1.58 points to the low-ability student and 1.75 points to the high-ability student) and Mrs. T (1.42 points to the low-ability student and 1.58 points to the high-ability student), met the two criteria and were selected as representative teachers. Below, we report those teachers' underlying perceptions and self-reported practices and examine how their perceptions and beliefs were aligned with the observed classroom results.

4.5.1. Mrs. B

Mrs. B's low-ability students experienced higher intellectual quality than did her high-ability students. Her expectations and perceptions of the low-ability students and her self-reported teaching practices follow this pattern. Lily, a low-ability student, may not be promoted to the fifth grade; Mrs. B did not lower the standards and continued to hold high and positive expectations for her student:

> I don't like to lower my standards, but I like the children to have some type of success also. So if I grade her on the level that I grade some of the other children, then she would never have any success. It is just that when she gives me an assignment, I make a judgment based on what I know she can do grades-wise.

Mrs. B also identified and recognized Lily's progress across semesters. At the beginning of the year, Lily "would give up immediately and say she can't do it….So I think she is starting to catch on a little faster." Mrs. B's accommodations, based on her understanding of this student, allowed Lily to have some success in mathematics. According to Mrs. B, Lily did not know the basic mathematical facts and abstract concepts very well, while "concrete things work better for her." Lily's strength was that she knew how to ask for help and was persistent. Thus, Mrs. B knew that Lily should not be assigned independent work and the best strategies for teaching her were to “pull her to the back of the room and work with her one on one” and to use explicit instruction (i.e., “show her an example of what we are doing and take her through them step by step”).

Importantly, Mrs. B did not attribute Lily’s non-successful grade promotion to the student despite her having some special needs. Instead, Mrs. B attributed Lily’s academic struggles to the apparent lack of parental support. Lily lives in a single-parent family, with her mother and grandmother, neither of whom has checked her school performance: “She [Lily] doesn’t have the parental support. I haven’t met the parents yet. It is May, no one has picked up any of her report cards.”

4.5.2. Mrs. R

Mrs. R is a teacher from an effective school whose low-ability student experienced greater intellectual quality than did a high-ability student. Amelia is a very shy, anxious, and less confident student. Mrs. R did not have low expectations of her; instead, she stated that “she has done well—she really has” and “she also tends to do on tests pretty much the way I expect her to.”But Amelia is shy and “doesn’t have that confidence yet in math.” Her teacher gave her the chance to present a calendar, during which “she is coming out a little bit, and I think that is going to give her the confidence she needs when she is doing her exams and when she is doing her paper.” Mrs. R also attempted to build Amelia’s confidence through verbally encouraging and creating a safe learning environment: “I tell her to calm down and breathe. You’ll do well, and if you don’t do well, then we know which ones we have to target next week.” Mrs. R also tried to use handy resources to better support Amelia’s learning of multiplication facts, which are a weakness. Mrs. R made multiplication rap songs, little math puzzles, flashcards, multiplication computer games, and Saturday tutoring to help her. Amelia gradually grasped the basic mathematical facts that she had completely lost previously.

4.5.3. Mrs. A

Mrs. A taught in a typical school where her low-ability student experienced lower intellectual quality than did her high-ability student. When asked about the reason for Mia, a low-ability female student’s failure to meet her expectations in math, Mrs. A responded:

> Some children are just not good at math, and it runs in the family. I talked to her mother about it [and] she said: “You know [Mrs. A], I was never good at math.” So they project that down through the generations, you know.

Mrs. A’s perceptions reflected that she treated students’ low performance as passed between generations and rooted in parents’ poor mathematics performance. When asked about Mia’s strengths in math, Mrs. A responded: “Those kinds of questions, I can’t answer right off the bat. You know, she’s [an] average student and she just does the best that she can.” Different from her effective schools colleagues, who identified their students’ strengths and weaknesses in math, Mrs. A does not seem to have a good understanding of her students. Her perceptions suggest that she thinks there is nothing that she can do. Asked whether there were any ways of teaching that she adopted for her low-ability student, Mrs. A responded: “No. Not other than what I give to the whole class.” She provided the same responses and asked if there were any ways of teaching that she just would not use with Mia (“No. I’m a generalizing person”). Mrs. A’s perceptions of Mia’s poor performance in math would seem to make it less likely that she would adopt individualized instruction and/or any other adaptation based on Mia’s weaknesses and strengths in mathematics.

Surprisingly, Mrs. A also mentioned that her students did not have supportive home learning environments, noting that Mia came from a single-parent family with four to five children and that her mother also had drug abuse problems at some points.

4.5.4. Mrs. T

When Mrs. T talked about Kevin, her low-ability student, she seemed to face pressure to help this student pass the fourth-grade tests. Mrs. T knew, from Kevin's mother, that she had drug abuse problems when he was born, so Kevin may have learning disabilities and problems staying focused. However, his mother was first in denial and refused to test Kevin, which did not help him get the support that he needed:

> Um…but, she does realize that he has some learning difficulties. And he's so immature. You know, he was okay for these two days. But, I mean, he can really just…uh! Like…he's on the level of a kindergartner or a first-grader. You know, just everything is funny, and he's laughing and giggling.

In the two days of classroom observations, Kevin behaved well, but Mrs. T was not surprised, reporting: "I think he's a little scared of me." Giving Kevin the opportunities to work with his peer sometimes works, but not always: "Sometimes they do kind of tease each other, and I kind of squelch that as soon as they start. But, um…I don't know…he's uh! It's frustrating. It's really frustrating. But then again, I understand why he is the way he is." Based on Mrs. T's responses, she seemed very frustrated and overwhelmed by this low-ability student.

When asked about the kinds of assistance and resources that are available to help him, Mrs. T believed that a small setting might help, but she had a big class: "A much smaller setting. Not in a regular classroom, no. Not with 27, and see, we can go up to 33. Never with 33 people in a classroom. He would be lost." Also, Kevin was signed up for one-to-one tutoring and reinforcement with a retired teacher to keep him focused, but this mainly focused on literacy, not math.

Teachers in both the effective and the typical schools attributed students' low performance to family-related issues such as parent drug issues and family size. Unsupportive home learning environments were a common constraint and challenge for these teachers when teaching low-ability students who might need additional attention and instruction. However, there are some nuanced differences in teachers' perceptions, beliefs, and self-reported teaching practices. Teachers from effective schools held high and positive expectations of low-ability students and adopted individualized instructions that particularly met their learning needs; on the contrary, the teachers from typical schools held static perceptions about students' failure in math, such as that it passed from generation to generation.

5. Discussion

The present study found that students in effective schools experienced more intellectual support and greater overall intellectual quality of mathematics instruction than their peers in typical schools. Our observational evidence documents classroom environments with a strong academic focus and teachers with high expectations for their low-ability students (Clewell et al., 2007; Jesse et al., 2004; Price & Waxman, 2005; Teddlie & Reynolds, 2000). Classroom environments from effective schools provided greater intellectual support to students. Low-ability students in effective schools experienced better intellectual quality of their learning environment, characterized by high academic expectations, mutual respect and support for all students, and deeper engagement in mathematical knowledge than their high-ability peers. The analysis of four mathematics teachers' interviews confirmed and expanded on these results; that is, the two teachers from effective schools hold high and positive expectations for the low-ability students, and they could identify low-ability students' strengths and weaknesses in mathematics, use the knowledge to make accommodations to teach them, and make use of resources to better support their mathematical learning. The typical school's teachers seemed very pessimistic about teaching low-ability students. Mrs. A holds that a student's poor performance passes between generations and is more likely to adopt general, non-individualized instruction to teach the low-ability students. Mrs. T is more negative and overwhelmed when facing low-ability students who have learning difficulties, blaming uncooperative parents.

This study also found some commonalities in the self-reported challenges and constraints of teaching low-ability students between effective and typical schools. Teachers from both schools mentioned that students, especially low-ability students, had unsupportive family environments, parental drug issues, and large families. As a result, those students presented behavior problems, could not concentrate on classroom learning and schoolwork, and showed no interest in mathematical learning. Generally, these challenges are not fundamentally different from other studies conducted in Urban areas (Du Plessis & Mestry, 2019; Muijs, 2003). What is similar across effective and typical schools relates to aspects of specific contextual circumstances, such as home learning environments, while what is different is how teachers perceive, expect, and teach students in mathematical classrooms. This finding opens a potential direction for future research; that is, rather than merely focusing on what makes effective schools different from ineffective ones, we might also need to study how effective schools overcome the challenges and pressures caused by distressing outside environments to provide a better school learning environment for students.

This study makes substantial contributions to current research. In examining how classroom learning environments contributed to school effectiveness, we went beyond focusing solely on the teacher side, such as teacher behaviors (Muijs & Reynolds, 2000) or instructional quality (Jong et al., 2004). We focused on teachers and students and their mutual contribution to the construction of high-quality learning environments.

Second, we examined classroom learning environments within contexts of school effectiveness and student ability. Results show that low-ability students from effective schools experienced better overall intellectual quality from their teachers than their peers (low- and high-ability) from typical schools. Sinclair et al. (2010) found that students from urban neighborhoods are less likely to progress toward positive outcomes after leaving school. It is not yet clear whether effective schools are able to add significant value to students' mathematical achievement since this study does not examine students' academic achievement. Surprisingly, teachers in our effective schools seem to compensate for low-ability students' disadvantaged academic status by providing better classroom learning environments and engagement opportunities.

The current study has limitations. We relied on a relatively small amount of observation data for multiple-level analysis, limiting the generalizability of the findings. Although schools in urban areas (Du Plessis & Mestry, 2019) faced similar challenges, such as paying less attention to low-achieving students and a lack of parental participation in children's education, an updated understanding of effective schools in urban contexts using larger-scale data collection approaches is needed. Second, our qualitative results are limited to four representative teachers, illustrating how they perceived and taught students of varying ability levels. Future research should conduct more in-depth analyses to reveal diverse patterns in teachers' perceptions, expectations, and self-reported teaching practices across typical and effective schools.

The result of this study also has implications for educators, school leaders, and policymakers. This study revealed that teachers in highly effective schools supported low-ability students' mathematical learning by maintaining high expectations and adopting more individualized instructional strategies, even in the face of external challenges. The experiences of effective schools in addressing the needs of low-ability students from urban areas may provide valuable guidance for teachers in typical schools, as they face similar and often challenging circumstances. Therefore, fostering communication and collaboration among teachers across different schools is essential. Furthermore, Maden (2001) argued that in order to achieve and sustain improvement, schools in urban areas must go beyond what might be termed as "normal efforts." Teachers from both school types faced extremely serious challenges that extend beyond the capacity of individual teachers. Consequently, improving schools in the urban areas requires collaboration between schools, parents, and the broader community.

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Conflict of Interests

The authors declare no conflict of interest.

Data Availability

Data from this study are available upon reasonable request from the first author.

LLMs Disclosure

LLM tools were not used in the production of this article.

Supplementary Material

Supplementary material for this article is available online in the format provided by the author (unedited).

References

Bates, D. M., & Sarkar, D. (2007). *lme4: Linear mixed-effects models using S4 classes* [Computer software] (R Package Version, 0, 99875–99876). http://lme4.r-forge.r-project.org/book

Bedard, K., & Do, C. (2005). Are middle schools more effective? The impact of school structure on student outcomes. *Journal of Human Resources*, *40*(3), 660–682. https://doi.org/10.3368/jhr.XL.3.660

Bloom, B. (1956). *Taxonomy of educational objectives, the classification of educational goals*. Susan Fauer Company Inc.

Charalambous, C. Y., & Praetorius, A. K. (2018). Studying mathematics instruction through different lenses: Setting the ground for understanding instructional quality more comprehensively. *ZDM*, *50*(3), 355–366. https://doi.org/10.1007/s11858-018-0914-8

Cheng, Y. C. (2022). *School effectiveness and school-based management: A mechanism for development*. Routledge.

Clare, L., & Aschbacher, P. R. (2001). Exploring the technical quality of using assignments and student work as indicators of classroom practice. *Educational Assessment*, *7*(1), 39–59. https://doi.org/10.1207/S15326977EA0701_5

Clewell, B. C., Campbell, P. B., & Perlman, L. (2007). *Good schools in poor neighborhoods: Defying demographics, achieving success*. The Urban Institute.

Cobb, P., & Wilhelm, A. G. (2022). Classroom teaching and learning as the focus of improvement research. In D. J. Peurach, J. L. Russell, L. Cohen-Vogel, & W. Penuel (Eds.), *The foundational handbook on improvement research in education* (pp. 139–164). Rowman & Littlefield.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, *31*(3/4), 175–190. https://doi.org/10.1080/00461520.1996.9653265

Creemers, B., & Kyriakides, L. (2007). *The dynamics of educational effectiveness: A contribution to policy, practice and theory in contemporary schools*. Routledge.

Creswell, J. W., Plano Clark, V. L., Gutmann, M. L., & Hanson, W. E. (2003). Advanced mixed methods research designs. In A. Tashakkori & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 209–240). Sage.

Desimone, L. M., & Long, D. (2010). Teacher effects and the achievement gap: Do teacher and teaching quality influence the achievement gap between Black and White and high-and low-SES students in the early grades? *Teachers College Record*, *112*(12), 3024–3073.

Dowson, M., & McInerney, D. M. (2003). What do students say about their motivational goals? Towards a more complex and dynamic perspective on student motivation. *Contemporary Educational Psychology*, *28*(1), 91–113. https://doi.org/10.1016/S0361-476X(02)00010-3

Du Plessis, P., & Mestry, R. (2019). Teachers for rural schools–a challenge for South Africa. *South African Journal of Education*, *39*, 1–9. https://doi.org/10.15700/saje.v39ns1a1774

Dukmak, S. (2009). Ability grouping and teacher-students interaction among high and low achieving students in middle primary schools in the United Arab Emirates. *Journal of Faculty of Education*, *26*, 1–30.

Edmonds, R. R. (1979). *Discussion of the literature and issues related to effective schooling*. Center of Urban Studies, Harvard Graduate School of Education.

Engel, L. C., Holford, J., & Pimlott-Wilson, H. (2010). Effectiveness, inequality and ethos in three English schools. *International Journal of Sociology and Social Policy*, *30*(3), 140–154. https://doi.org/10.1108/01443331011033337

Fisher, D., Waldrip, B., & den Brok, P. (2005). Students' perceptions of primary teachers' interpersonal behavior and of cultural dimensions in the classroom environment. *International Journal of Educational Research*, *43*(1/2), 25–38. https://doi.org/10.1016/j.ijer.2006.03.004

Garrahy, D. A. (2001). Three third-grade teachers' gender-related beliefs and behavior. *The Elementary School Journal*, *102*(1), 81–94. https://doi.org/10.1086/499694

Jesse, D., Davis, A., & Pokorny, N. (2004). High-achieving middle schools for Latino students in poverty. *Journal of Education for Students Places at Risk*, *9*(1), 23–45. https://doi.org/10.1207/S15327671ESPR0901_2

Jong, R. D., Westerhof, K. J., & Kruiter, J. H. (2004). Empirical evidence of a comprehensive model of school effectiveness: A multilevel study in mathematics in the 1st year of junior general education in the Netherlands. *School Effectiveness and School Improvement*, *15*(1), 3–31.

Kenward, M. G., & Roger, J. H. (1997). Small sample inference for fixed effects from restricted maximum likelihood. *Biometrics*, 983–997.

Ketterlin-Geller, L. R., Chard, D. J., & Fien, H. (2008). Making connections in mathematics: Conceptual mathematics intervention for low-performing students. *Remedial and Special Education*, *29*(1), 33–45. https://doi.org/10.1177/0741932507309711

Lemke, J. L. (2001). Articulating communities: Sociocultural perspectives on science education. *Journal of Research in Science Teaching*, *38*(3), 296–316.

Maden, M. (Ed.). (2001). *Success against the odds—Five years on: Revisiting effective schools in damaged areas*. London: Routledge; Falmer.

Merritt, E. G., Palacios, N., Banse, H., Rimm-Kaufman, S. E., & Leis, M. (2017). Teaching practices in Grade 5 mathematics classrooms with high-achieving English learner students. *The Journal of Educational Research*, *110*(1), 17–31. https://doi.org/10.1080/00220671.2015.1034352

Muijs, D. (2003). Improving and effective schools in disadvantaged areas: A summary of research findings. *Improving Schools*, *6*(1), 5–11.

Muijs, D., Harris, A., Chapman, C., Stoll, L., & Russ, J. (2004). Improving schools in socioeconomically disadvantaged areas: A review of research evidence. *School Effectiveness and School Improvement*, *15*(2), 149–175. https://doi.org/10.1076/sesi.15.2.149.30433

Muijs, D., & Reynolds, D. (2000). School effectiveness and teacher effectiveness in mathematics: Some preliminary findings from the evaluation of the mathematics enhancement programme (primary). *School Effectiveness and School Improvement*, *11*(3), 273–303.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all*.

Newmann, F. M. (1996). Authentic assessment in social studies: Standards and examples. In G. D. Phye (Ed.), *Handbook of classroom assessment: Learning, achievement, and adjustment* (pp. 359–380). Academic Press.

Newmann, F. M., & Weglage, G. G. (1993). Five standards for authentic instruction. *Educational Leadership*, *50*(7), 8–12.

Newmann, F. M., Marks, H. M., & Gamoran, A. (1996). Authentic pedagogy and student performance. *American Journal of Education*, *104*(4), 280–312.

Nurmi, J. E. (2012). Students' characteristics and teacher–child relationships in instruction: A meta-analysis. *Educational Research Review*, *7*(3), 177–197. https://doi.org/10.1016/j.edurev.2012.03.001

Osborne, M. C. (2021). Teacher instructional practices and student mathematics achievement. *Journal of Educational Research and Practice*, *11*(1), 345–358. https://doi.org/10.5590/JERAP.2021.11.1.25

Pelemo, P. E. K. (2022). *Parental involvement: Investigating the roles, barriers, and strategies of effective parent-teacher relationships in middle school math* [Unpublished doctoral dissertation]. Southeastern University.

Price, D. D., & Waxman, H. C. (2005). Effective schools. In V. A. Anfara Jr., G. Andrews, & S. B. Mertens (Eds.), *The encyclopedia of middle grades education* (pp. 185–188). National Middle School Association and Information.

Purkey, S. C., & Smith, M. S. (1983). Effective schools: A review. *The Elementary School Journal*, *83*(4), 427–452. https://doi.org/10.1086/461325

Raham, H. (2004). *Schools that make a difference: Twelve Canadian secondary schools in low-income settings* [Paper presentation]. The 2004 Annual Conference of the American Educational Research Association, San Diego, CA, USA.

Reynolds, D. (2010). *School effectiveness*. A&C Black.

Riegle-Crumb, C., & Humphries, M. (2012). Exploring bias in math teachers' perceptions of students' ability by gender and race/ethnicity. *Gender & Society*, *26*(2), 290–322. https://doi.org/10.1177/0891243211434614

Scheerens, J. (2016). *Educational effectiveness and ineffectiveness: A critical review of the knowledge base*. Springer.

Secada, W. G., & Lee, O. (2000). *Highly effective USI schools: An outlier study*. University of Wisconsin.

Sinclair, S., McKendrick, J. H., & Scott, G. (2010). Failing young people? Education and aspirations in a deprived community. *Education, Citizenship and Social Justice*, *5*(1), 5–20. https://doi.org/10.1177/1746197909353564

Smith, J. B., Lee, V. E., & Newmann, F. M. (2001). *Instruction and achievement in Chicago elementary schools: Improving Chicago's schools*. Consortium on Chicago School Research.

Smith, T. M., Desimone, L. M., & Ueno, K. (2005). "Highly qualified" to do what? The relationship between NCLB teacher quality mandates and the use of reform-oriented instruction in middle school mathematics. *Educational Evaluation and Policy Analysis*, *27*(1), 75–109.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership*, *61*(5), 12–17.

Straehler-Pohl, H., Fernández, S., Gellert, U., & Figueiras, L. (2014). School mathematics registers in a context of low academic expectations. *Educational Studies in Mathematics*, *85*(2), 175–199. https://doi.org/10.1007/s10649-013-9503-5

Stylianides, A. J., & Stylianides, G. J. (2007). Learning mathematics with understanding: A critical consideration of the learning principle in the principles and standards for school mathematics. *The Mathematics Enthusiast*, *4*(1), 103–114. https://doi.org/10.54870/1551-3440.1063

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2015). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*, *18*, 123–140. https://doi.org/10.1007/s10857-014-9279-2

Sullivan, P., & Mornane, A. (2014). Exploring teachers' use of, and students' reactions to, challenging mathematics tasks. *Mathematics Education Research Journal*, *26*(2), 193–213. https://doi.org/10.1007/s13394-013-0089-0

Tan, C. Y., & Dimmock, C. (2022). The relationships among between-class ability grouping, teaching practices, and mathematics achievement: A large-scale empirical analysis. *Educational studies*, *48*(4), 471–489. https://doi.org/10.1080/03055698.2020.1780416

Teddlie, C., Kirby, P. C., & Stringfield, S. (1989). Effective versus ineffective schools: Observable differences in the classroom. *American Journal of Education*, *97*(3), 221–236.

Teddlie, C., & Reynolds, D. (2000). *The international handbook of school effectiveness research*. Psychology Press.

Tiedemann, J. (2002). Teachers' gender stereotypes as determinants of teacher perceptions in elementary school mathematics. *Educational Studies in Mathematics*, *50*(1), 49–62. https://doi.org/10.1023/A:1020518104346

Waxman, H. C., Padrón, Y. N., & Garcia, A. (2007). Educational issues and effective practices for Hispanic students. In S. J. Paik & H. J. Walberg (Eds.), *Narrowing the achievement gap: Strategies for educating Latino, Black and Asian students* (pp. 131–151). Springer.

Waxman, H. C., Padrón, Y. N., & Keese, J. (2021). Learning environment and students' classroom behavior differences between effective, average, and ineffective urban elementary schools for Hispanic students. *Educational Research for Policy and Practice*, *20*, 307–324. https://doi.org/10.1007/s10671-020-09281-7

Wimer, J. W., Ridenour, C. S., Thomas, K., & Place, A. W. (2001). Higher order teacher questioning of boys and girls in elementary mathematics classrooms. *The Journal of Educational Research*, *95*(2), 84–92. https://doi.org/10.1080/00220670109596576

Yu, R., & Singh, K. (2018). Teacher support, instructional practices, student motivation, and mathematics achievement in high school. *The Journal of Educational Research*, *111*(1), 81–94. https://doi.org/10.1080/00220671.2016.1204260

Zedan, R. (2010). New dimensions in the classroom climate. *Learning Environments Research*, *13*(1), 75–88. https://doi.org/10.1007/s10984-009-9068-5

Zhu, Y., & Kaiser, G. (2022). Impacts of classroom teaching practices on students' mathematics learning interest, mathematics self-efficacy and mathematics test achievements: A secondary analysis of Shanghai data from the international video study Global Teaching InSights. *ZDM*, *54*(3), 581–593. https://doi.org/10.1007/s11858-022-01343-9

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